

Light quark mass ratio from Dalitz plot of $\eta \rightarrow \pi^+\pi^-\pi^0$ decay

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High statistics Dalitz-plot distribution of $\eta \rightarrow \pi^+\pi^-\pi^0$ decay obtained recently by KLOE collaboration [1] is fitted to the results of corresponding theoretical calculations in Chiral Perturbation Theory (ChPT) with unitarity corrections taken into account. The quark mass ratio $Q = \sqrt{(m_s^2 - (m_d + m_u)^2/4)/(m_d^2 - m_u^2)}$ can be obtained from this analysis. We get $Q = 22.8 \pm 0.4$ which differs from the value $Q_{DT} = 24.2$ that follows from Dashen's theorem and agrees with recently calculated electromagnetic kaon mass difference.

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The possibility to extract light quark mass difference from $\eta \rightarrow \pi^+\pi^-\pi^0$ decay is known for a long time [2]. In ChPT the decay width Γ depends on quark mass ratios and theoretically calculable factor $\bar{\Gamma}$ [3]:

$$\Gamma = \left(\frac{Q_{DT}}{Q}\right)^4 \bar{\Gamma}, \quad (1)$$

where

$$Q^{-2} = \frac{m_d^2 - m_u^2}{m_s^2 - \hat{m}^2}, \quad \hat{m} = \frac{m_d + m_u}{2}, \quad (2)$$

m_u, m_d, m_s - are up, down and strange quark masses,

$$Q_{DT}^{-2} = \frac{((m_{K^0}^2 - m_{K^+}^2) - (m_{\pi^0}^2 - m_{\pi^+}^2)) m_{\pi^0}^2}{(m_K^2 - m_{\pi^0}^2) m_K^2} = (24.2)^{-2}, \quad (3)$$

with $m_K^2 = (m_{K^+}^2 + m_{K^0}^2 - m_{\pi^+}^2 + m_{\pi^0}^2)/2$.

Note that $Q_{DT} = Q$ if electromagnetic mass differences of kaons and pions are equal to each other as Dashen's theorem states [4]. Experimental (Particle Data Group) value $\Gamma = 291 \pm 21$ eV [5] is far from one-loop ChPT value $\bar{\Gamma} = 167 \pm 50$ eV [3] and from the values $\bar{\Gamma} = 209 \pm 20$ eV [6] and $\bar{\Gamma} = 219 \pm 22$ eV [7] obtained with higher order corrections taken into account by dispersion method. In [6] the subtraction polynomial was taken from the decomposition of one-loop order amplitude and had therefore uncertainties connected to higher orders ChPT corrections. These uncertainties were further fixed [8] by fitting the experimental data [9] on Dalitz-plot distribution in the decay considered: $\bar{\Gamma} = 213_{-12}^{+3}$ eV. There was a conjecture in [8] that new experimental data on Dalitz-plot distribution will give slightly different value of $\bar{\Gamma}$. Now these new experimental data (contradicting to the old ones [9]) are available [1]. In what follows we will use them to get new value of $\bar{\Gamma}$ and new value of quark mass ratio Q , as a consequence.

We use the method of work [8] and remind it here for completeness. In order to simulate the experimental Dalitz-plot distribution we take it in a form

$$1 + ay + by^2 + cy^3 + dx^2 \quad (4)$$

with $a = -1.075 \pm 0.008$, $b = 0.118 \pm 0.009$, $c = 0.13 \pm 0.02$, $d = 0.049 \pm 0.008$ [1] and y, x defined in a standard way

$$y = \frac{3T_0}{Q} - 1, \quad x = \frac{\sqrt{3}}{Q} (T_+ - T_-), \quad Q = T_+ + T_- + T_0,$$

T_+, T_-, T_0 are the kinetic energies of pions in the rest frame $\eta \rightarrow \pi^+\pi^-\pi^0$ decay. We divide the Dalitz plot in 10×10 bins ($x \times y$) that have equal number of events for the distribution considered. Then the number of events in each bin (n) is simulated by Gaussian distribution with variance equal to n . We used $n = 10000$ to get the full statistics $N = 100n = 1000000$ like in the experiment [1]. From theoretical point of view the amplitude of $\eta \rightarrow \pi^+\pi^-\pi^0$ decay have an approximate solution from Eq.(5.28) of [6]. It contains the subtraction polynomial

$$P(s) = \alpha + \beta s_a + \gamma s_a^2 + \delta(s_b - s_c)^2, \quad (5)$$

where s_a, s_b and s_c are invariant masses squared of $\pi^+\pi^-, \pi^+\pi^0$ and $\pi^-\pi^0$ pairs, respectively. For the values of parameters α, β, γ and δ within the regions

$$\begin{aligned} \alpha &= -1.28 \pm 0.14, & \beta &= 21.81 \pm 1.52 \text{ GeV}^{-2} \\ \gamma &= 4.09 \pm 3.18 \text{ GeV}^{-4}, & \delta &= 4.19 \pm 1.08 \text{ GeV}^{-4} \end{aligned} \quad (6)$$

(the case of zero subtraction points [6]) the "Minuit" fit of above simulated experimental Dalitz-plot distribution has terminated on the values

$$\begin{aligned} \alpha_0 &= -1.14, & \beta_0 &= 23.33 \text{ GeV}^{-2} \\ \gamma_0 &= 1.03q \text{ GeV}^{-4}, & \delta_0 &= 5.27 \text{ GeV}^{-4} \end{aligned} \quad (7)$$

with $\chi^2/Nd.o.f. = 152/(100 - 4)$.

Three from four parameters are at the boundary of allowed region (6). This probably means that the guess [6] of the size of allowed region should be changed. Equally possible is the fit with the scaled values of parameters α, β, γ and δ because the normalization factor of the amplitude is not defined by the Dalitz-plot distribution. In our case the scaling of parameters α, β, γ and δ puts them outside the allowed region (6) and no freedom in the scaling (no error in $\bar{\Gamma}$) is possible. This way we get $\bar{\Gamma} = 229$ eV what corresponds according to eqs. (1)-(3)

to the light quark mass ratio $Q = 22.8 \pm 0.4$. The errors here are due to the errors in the experimental value of the width Γ . So, we conclude, high statistics Dalitz-plot distribution gives the value of light quark mass ratio Q slightly lower than that from the assumption of equality of kaon and pion electromagnetic mass differences ($Q_{DT} = 24.2$). This is in agreement with calculations of electromagnetic mass differences for pions and

kaons [10, 11] which find large violations to Dashen's theorem ($Q = 22.0 \pm 0.6$). Our result agrees also very well with that of works [6] and [7], where the values $Q = 22.4 \pm 0.9$ and $Q = 22.7 \pm 0.8$ were obtained, correspondingly.

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